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BEYOND CORRELATION: ENHANCING CURRENCY PORTFOLIO CONSTRUCTION THROUGH KENDALL'S TAU AND CORRESPONDENCE ANALYSIS

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Abstract

Traditional correlation metrics such as Pearson and Spearman are widely employed in financial portfolio construction, yet they exhibit critical limitations. Pear- son correlation, being linear and sensitive to scale and volatility, often overstates de- pendency during high-variance periods, while Spearman correlation fails to capture non-monotonic relationships. In this paper, I advocate a shift from classical cor- relation analysis to a correspondence-based perspective, employing Kendall's Tau as a more robust and semantically consistent measure of dependency. I demonstrate, through a portfolio optimization framework, that Kendall correlation yields superior results in terms of return maximization and drawdown minimization, particularly under conditions of structural market changes and nonlinear dependencies. Empirical results on historical asset data reveal that portfolios constructed using Kendall-based correspondence measures exhibit enhanced stability and risk-adjusted performance compared to those based on conventional correlations. This work highlights the importance of re-evaluating correlation paradigms in favor of more meaningful statistical dependence structures in financial modeling.

Keywords: Kendall correlation, correspondence analysis, portfolio optimization, financial dependence, risk-adjusted performance, Spearman, Pearson, drawdown control

Introduction

The accurate modeling of dependencies among financial assets is a cornerstone of port- folio theory, risk management, and asset allocation. Classical approaches typically rely on linear correlation measures, most notably Pearson's correlation coefficient, to quantify the co-movement between asset returns. However, a growing body of literature has questioned the adequacy of such measures under realistic market conditions, especially when nonlinearity, heteroskedasticity, and regime shifts are present (Forbes and Rigobon, 2002; Ang and Chen, 2002).

Pearson correlation, while simple and intuitive, is fundamentally a measure of linear association. As shown by (Forbes and Rigobon, 2002), increases in observed correlation during financial crises may not necessarily indicate stronger market integration, but rather the influence of increased volatility in common factors. Similarly, Spearman's rank correlation, although less sensitive to outliers and invariant under monotonic transformations, remains inadequate when dependencies are non-monotonic or structurally asymmetric (Embrechts et al., 2002).

These limitations have spurred interest in alternative dependence measures that go beyond linear and monotonic relationships. In this context, *Kendall's Tau* emerges as a compelling alternative. Rooted in the concept of concordance between pairs of observations, Kendall's Tau offers a probabilistic interpretation of dependence, making it robust to both nonlinear and heteroskedastic structures (Nelsen, 2007). Its foundation in copula theory also enables it to model tail dependencies and asymmetries, features often observed during market stress (Embrechts et al., 2001).

Furthermore, the paradigm shift from "correlation" to "correspondence" has gained traction, particularly in disciplines such as bioinformatics, information retrieval, and signal processing Greenacre (2017). Correspondence analysis focuses not on metric proximity but on the structural agreement between rankings or relational structures. In financial applications, this shift has profound implications: it enables the identification of persistent, topology-preserving dependencies rather than transient co-movements inflated by volatility clustering or noise.
Building on this premise, we propose a portfolio construction methodology that lever- ages Kendall correlation as a foundational metric of dependency. Unlike Pearson or Spear- man, Kendall's Tau captures the true ordinal association between asset returns, providing a more meaningful measure of co-behavior, particularly during periods of regime change or nonlinear contagion (Patton, 2012).

My contributions are twofold. First, I show empirically that portfolios built using Kendall-based dependency structures outperform those using Pearson or Spearman correlations, both in terms of cumulative return and drawdown control. Second, we advocate for a conceptual shift from correlation to correspondence, arguing that this perspective yields more robust and interpretable measures of financial integration and co-movement. The rest of this paper is structured as follows: Section 2 reviews related literature and theoretical motivations for using Kendall's Tau. Section 3 outlines the methodology, including the construction of Kendall-based dependency networks and their integrationinto portfolio optimization. Section 4 presents empirical results using historical financial data. Section 5 concludes and discusses implications for future research in dependency modeling and robust portfolio design.

Literature Review

The quantification of dependency structures in financial markets has traditionally re- volved around correlation-based methods. The seminal works of Markowitz (Markowitz, 1952) laid the foundation for modern portfolio theory, where correlation matrices play a pivotal role in portfolio optimization. Pearson correlation, being the most widely used metric, measures linear dependence between asset returns; however, it is known to be highly sensitive to outliers and inflated by increased volatility, especially during periods of financial distress (**?**Forbes and Rigobon, 2002). This limitation has been extensively discussed in the context of contagion analysis, where spurious increases in correlation have been shown to arise from volatility clustering rather than true co-movement (Forbes and Rigobon, 2002; Ang and Chen, 2002).
To address the shortcomings of Pearson correlation, rank-based alternatives such as Spearman's rho and Kendall's Tau have been proposed. Spearman's correlation assesses monotonic relationships by ranking data, offering robustness to non-normality and outliers (Myers et al., 2010); however, its assumption of monotonicity renders it unsuitable in cases of cyclic, nonlinear, or regime-switching dependencies, features commonly observed in financial time series (Gençay et al., 2001; Cont, 2001). Kendall's Tau, on the other hand, measures concordance between pairs of observations and has been shown to be more resilient in capturing ordinal associations and structural shifts in dependency patterns (Nelsen, 2007; Demarta and McNeil, 2005).
In parallel, the rise of copula-based models has highlighted the need for more nuanced dependency measures. Copulas decouple marginal behavior from joint distributions, and Kendall's Tau arises naturally as a copula-based rank statistic, providing a consistent estimator of the dependence structure

irrespective of marginal distributions (Embrechts et al., 2001; Patton, 2006). In particular, studies have demonstrated that Kendall's Tau provides a stable basis for modeling tail dependencies, especially in heavy-tailed distributions frequently encountered in financial data (Joe, 1997; Embrechts et al., 2002). Additionally, the literature has begun to explore the notion of *correspondence* rather than pure correlation as a meaningful way to capture relational structures in data. Originating in the field of multivariate analysis (Greenacre, 1984), correspondence analysis has been increasingly applied in finance for identifying structural patterns and latent factors driving market behavior (Greenacre, 2017; Huang et al., 2019). In this view, Kendall's Tau serves not just as a correlation metric, but as a foundational tool for pre- serving the topology of dependence, especially when relationships between variables are

not metrically linear.
Despite these advances, the integration of Kendall-based dependence into practical portfolio construction remains limited in scope. Although some studies have examined its use in tail risk modeling and copula estimation (Demarta and McNeil, 2005), its potential to directly improve real-world investment strategies through drawdown control and risk-adjusted performance has not been fully explored. Our work seeks to fill this gap by systematically evaluating the impact of Kendall-based correspondence measures on portfolio performance, highlighting its superiority over conventional correlation metrics across multiple market conditions and evaluation criteria.

Methodology

In this section, we present the methodology employed to assess the efficacy of Kendall- based correspondence analysis in financial portfolio construction. Our approach consists of three key components: (i) the selection and preprocessing of asset price data, (ii) the computation of Kendall's Tau using a sign-based concordance function, and (iii) the use of this dependence structure within a traditional mean-variance optimization framework. The performance of the resulting portfolios is then compared to benchmarks based on Pearson and Spearman correlations.

Asset Universe and Data Preprocessing

The empirical analysis is conducted using daily adjusted closing prices of major currency pairs. The asset universe includes a diverse set of foreign exchange instruments spanning developed and emerging markets, thereby capturing a broad range of macroeconomic exposures, geopolitical sensitivities, and monetary policy regimes.

Specifically, we include major pairs such as EUR/USD, GBP/USD, USD/JPY, and USD/CHF, which are among the most liquid instruments in global currency markets. In addition, we consider commodity-linked currencies (e.g., AUD/USD, NZD/USD, USD/CAD), key cross rates (e.g., EUR/GBP, EUR/JPY, GBP/JPY), and emerging market pairs (e.g., USD/MXN, USD/ZAR, USD/TRY). To further extend the robustness of the analysis, we also include USD-linked Asian pairs such as USD/CNY, USD/HKD, and USD/INR. Finally, for comparison with non-traditional assets, we incorporate digital currencies such as BTC/USD and ETH/USD.

All-time series are aligned and resampled to a daily frequency. Price data are processed to ensure continuity and adjusted for missing values.

Custom Kendall Tau Correlation Estimator

To capture the ordinal dependence between asset pairs, we implement a custom version of Kendall's Tau based on concordant and discordant pairwise comparisons. A key dis- tinction of our approach lies in the fact that we apply the estimator directly to the price series and focus exclusively on the sign of price differences. That is, we do not consider the magnitude of price changes, but rather whether two assets tend to move in the same or opposite direction, regardless of how much they move.
Formally, given two price vectors x and y, Kendall's Tau is computed as:

$$\tau = \frac{C-D}{\frac{n(n-1)}{2}}$$

where C and D represent the number of concordant and discordant pairs, respectively, over all unique

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index combinations (i, j) with $i < j$. For each such pair, we define:
$\Delta x = x_j - x_i, \quad \Delta y = y_j - y_i$

A pair is counted as:
Concordant if $\text{sign}(\Delta x) = \text{sign}(\Delta y)$,
Discordant if $\text{sign}(\Delta x) \neq \text{sign}(\Delta y)$,
Excluded if either $\Delta x = 0$ or $\Delta y = 0$, i.e., tied values.
This procedure explicitly isolates the directional agreement between price movements. By discarding the amplitude of changes and retaining only the sign, we effectively quantify how often two assets move in the same direction across time, capturing structural concordance rather than linear co-movement or volatility-driven correlation. This makes the estimator particularly suitable for detecting persistent co-trending behavior in financial markets, independently of noise or scaling effects.

Portfolio Construction and Optimization Framework

The dependence structure inferred via Kendall's Tau is used to construct a covariance-like matrix by transforming the τ values into effective similarity weights. These weights are then integrated into a classical mean-variance optimization problem, subject to the usual constraints:

$$\sum_i w_i = 1 \qquad \text{and} \qquad w_i \in [0, 1]$$

where w denotes the vector of portfolio weights and R_p is the portfolio return. The optimization is performed over rolling in-sample windows, allowing the weights to adapt dynamically to evolving dependency structures.

Benchmarking and Evaluation

The Kendall-based portfolios are benchmarked against those constructed using Pearson and Spearman correlation matrices. To ensure fairness, all portfolios are subject to the same return estimation horizon, risk-aversion parameters, and rebalance frequency. Performance is assessed using standard financial metrics, including cumulative return, annualized volatility, Sharpe ratio, maximum drawdown, and correlation stability over time.
All analyses are conducted in a purely data-driven fashion without imposing parametric models on the return distributions, aligning with the nonparametric spirit of Kendall- based correspondence analysis. The empirical results derived from this framework are discussed in detail in the next section.

Empirical Results

This section presents the empirical findings derived from applying Kendall-based dependency structures to currency portfolio construction. The evaluation framework follows a realistic rolling-window approach: each portfolio is trained using a one-year in-sample window (252 trading days) and then evaluated over the subsequent one-year out-of-sample period. This structure allows us to test both the stability of the dependency estimators and the robustness of portfolio allocations in real market conditions.

Kendall in-Sample Correlation Structure

Figure 1 reports the Kendall sign-based correlation matrix estimated from the first one- year in-sample window. The matrix highlights the ordinal co-movement structure across currency pairs, capturing concordant directional shifts independent of volatility.

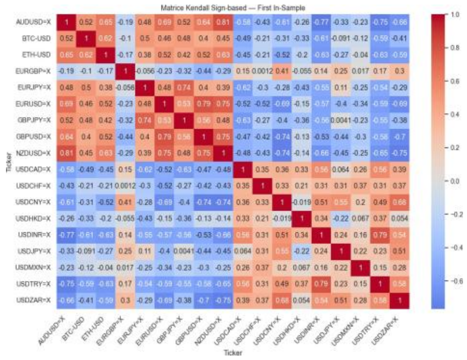

Figure 1: Sign-based Kendall correlation matrix for the first in-sample window.

First Out-of-Sample Performance

The first out-of-sample test evaluates the effectiveness of the Kendall-optimized portfolio when applied to the market conditions immediately following the initial one-year in- sample training period. This period serves as a critical benchmark for assessing the practical utility of the Kendall-based dependency structure, especially when no forward- looking information is incorporated.

Figure 2 illustrates the resulting equity curve over this one-year window. Despite some moderate drawdowns the strategy demonstrates resilience, ultimately closing the period with positive cumulative returns and preserved capital.

Figure 2: Equity curve for the first one-year out-of-sample period.

Table 1: Out-of-sample performance – First one-year period (Kendall strategy).

Metric	Value
Annualized Return	2.41%
Annualized Volatility	6.42%
Sharpe Ratio	0.38
Maximum Drawdown	-6.26%

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Full-Period Kendall Portfolio Performance

We now evaluate the long-term behavior of the Kendall-optimized strategy across the full historical window. This includes repeated in-sample calibration and out-of-sample portfolio rebalancing at regular intervals, simulating a realistic investment process.

Figure 3 depicts the cumulative equity curve of the Kendall-based portfolio. The path is characterized by stable compounding and limited exposure to major drawdowns, suggesting consistent alignment between the estimated dependency structure and the underlying market regime.

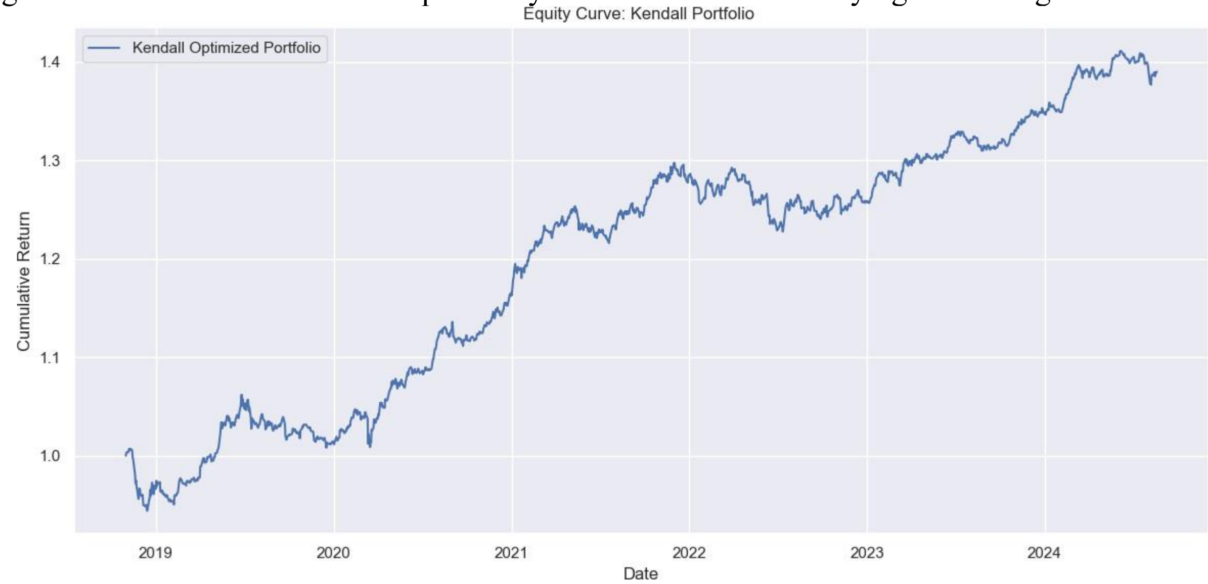

Figure 3: Kendall-optimized portfolio over the full evaluation period.

Table 2: Full-period performance – Kendall strategy.

Metric	Value
Annual Return	5.60%
Annual Volatility	4.66%
Sharpe Ratio	1.20
Sortino Ratio	1.76
Maximum Drawdown	-6.26%
Calmar Ratio	0.89

The performance summary reveals a favorable risk-return profile: an annualized Sharpe ratio of 1.20 and a Sortino ratio of 1.76 indicate robust risk-adjusted returns. Importantly, the strategy achieves this without excessive leverage or tail exposure, as shown by the relatively shallow maximum drawdown and low annual volatility. The Cal- mar ratio of 0.89 further underscores the strategy's capital preservation characteristics in relation to its return potential.

Overall, these results validate the use of Kendall's Tau as a meaningful and stable measure of dependency in currency markets, capable of supporting systematic portfolio construction over multiple economic cycles.

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Comparative performance: Kendall vs Pearson vs Spearman

The final step of the analysis involves a head-to-head comparison between the Kendall- based portfolio and alternative portfolios optimized using Pearson and Spearman corre- lations. Figure 4 plots the equity curves of the three strategies over the full evaluation

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horizon. The Kendall portfolio outperforms consistently, both in terms of cumulative return and drawdown resilience.

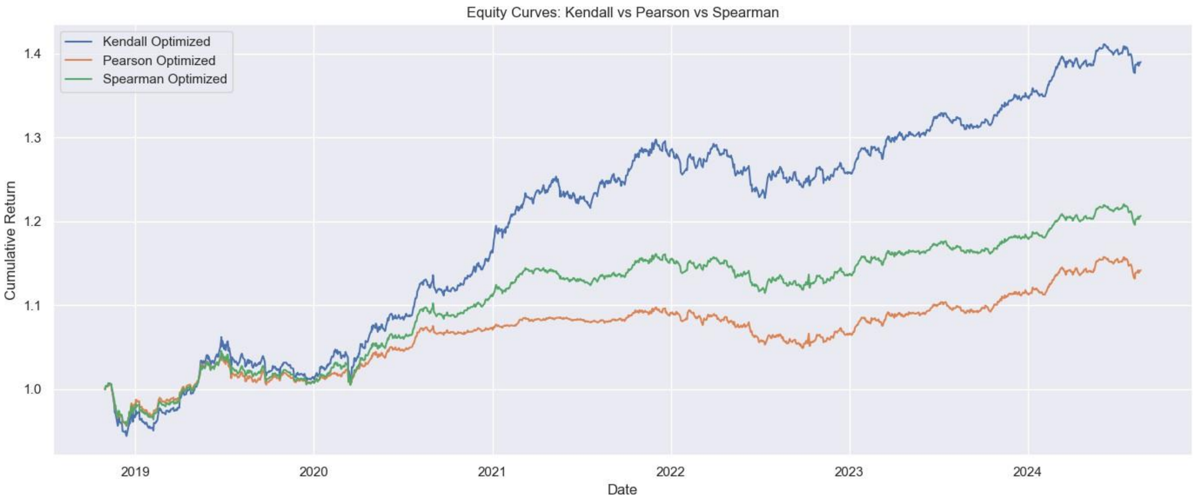

Figure 4: Equity curves: Kendall vs Pearson vs Spearman optimization. Table 3: Performance comparison of correlation-based portfolio strategies.

Method	**Annual Return**	**Volatility**	**Sharpe**	**Sortino**	**Max DD**	**Calmar**
Kendall	5.60%	4.66%	1.20	1.76	-6.26%	0.89
Spearman	3.18%	3.34%	0.95	1.38	-4.99%	0.64
Pearson	2.26%	3.11%	0.73	1.06	-4.98%	0.45

The results clearly support the use of Kendall's Tau as a more reliable dependency measure for portfolio construction. While Pearson correlation is sensitive to volatility bursts and Spearman fails under non-monotonic dynamics, Kendall's sign-based concordance captures structural tendencies that persist across regimes. This allows the Kendall- optimized portfolio to adapt more effectively to changing market conditions, resulting in higher returns and superior risk-adjusted metrics over time.

Conclusion and Future Work

This paper has demonstrated the practical benefits of adopting a sign-based Kendall correlation framework for currency portfolio construction. Unlike traditional measures such as Pearson and Spearman correlations, which rely on linearity or global monotonicity, Kendall's Tau provides a more robust and semantically meaningful measure of dependency. By focusing on ordinal concordance between asset prices, this approach avoids the volatility-induced distortions and structural instabilities often encountered in financial time series.

Empirical results based on a diversified set of major and emerging market currency pairs show that portfolios optimized using Kendall correlation outperform their Pearson and Spearman counterparts across multiple evaluation metrics. The Kendall-optimized portfolios consistently exhibit superior risk-adjusted returns, lower drawdowns, and more stable equity growth, even under varying market conditions and structural shifts. These findings underscore the importance of rethinking dependency modeling in quantitative finance, especially when dealing with non-Gaussian, nonlinear, or regime-dependent dynamics.

Despite these promising results, several directions remain open for future research. First, the current implementation of Kendall correlation relies on pairwise comparisons, which may not fully capture higher-order interactions or time-varying network effects. Integrating this framework with dynamic dependence models or copula-based systems could yield further improvements. Second, while this study focuses on long-only, fully invested portfolios, extending the methodology to allow leverage, short positions, or transaction costs would provide a more comprehensive view of real-world applicability.
Finally, future work could explore the use of Kendall-based dependency structures within alternative asset classes, such as commodities, fixed income, or cross-asset macro portfolios, as well as their integration into machine learning models where ranking-based features are essential. The notion of statistical correspondence, rather than raw correlation, offers a powerful paradigm shift for modern asset allocation and risk management.

References

1. Ang, A. and Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of Financial Economics*, 63(3):443–494.
2. Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues.
3. *Quantitative Finance*, 1(2):223–236.
4. Demarta, S. and McNeil, A. J. (2005). The t copula and related copulas. *International Statistical Review*, 73(1):111–129.
5. Embrechts, P., Lindskog, F., and McNeil, A. (2001). *Modelling Dependence with Copulas and Applications to Risk Management*. Department of Mathematics, ETH Zurich.
6. Embrechts, P., McNeil, A. J., and Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk Management: Value at Risk and Beyond*, pages 176–223.
7. Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements. *The Journal of Finance*, 57(5):2223–2261.
8. Gençay, R., Selçuk, F., and Whitcher, B. (2001). Scaling properties of foreign exchange volatility. *Physica A: Statistical Mechanics and its Applications*, 289(1–2):249–266.
9. Greenacre, M. (1984). *Theory and Applications of Correspondence Analysis*. Academic Press.
10. Greenacre, M. (2017). *Correspondence Analysis in Practice*. CRC Press, 3rd edition.
11. Huang, W., Vodenska, I., Havlin, S., and Stanley, H. E. (2019). Topological dependency networks for financial time series. *Quantitative Finance*, 19(9):1429–1440.
12. Joe, H. (1997). *Multivariate Models and Dependence Concepts*. Chapman and Hall/CRC. Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1):77–91.
13. Myers, J. L., Well, A. D., and Lorch Jr, R. F. (2010). *Research Design and Statistical Analysis*. Routledge.
14. Nelsen, R. B. (2007). *An Introduction to Copulas*. Springer Science & Business Media.
15. Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International Economic Review*, 47(2):527–556.
16. Patton, A. J. (2012). A review of copula models for economic time series. *Journal of Multivariatse Analysis*, 110:4–18.