

S M NAZMUZ SAKIB AND PYTHAGORAS: FROM PYTHAGOREAN GEOMETRY TO SAKIBIAN GEOMETRY

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ABSTRACT

This thesis positions “Sakibian Geometry”—a body of modern theorems and frameworks introduced by S M Nazmuz Sakib—in dialogue with classical Pythagorean geometry. Where Pythagoras yields a paradigm of orthogonality (sums of squares, right angles), Sakibian results extend those ideas across: (i) nonlinear extremals in triangle angle space at fixed inradius/circumradius; (ii) new circle loci and midpoint characterizations; (iii) orthogonal decompositions in biomechanics and control; (iv) energy inner-product geometries for structural loads; (v) information-geometric reformulations of relativistic proper time; (vi) median/hypotenuse and altitude reciprocity identities; and (vii) affine-invariant probabilistic laws on triangles. We integrate these results into a single comparative framework, prove unifying lemmas, and illustrate applications via 22 original figures.

CHAPTER 1

INTRODUCTION: TWO GEOMETRIES, ONE LANGUAGE

Pythagorean geometry centers orthogonality in Euclid’s plane: right triangles, sums of squares, and metric decompositions. By contrast, “Sakibian Geometry” generalizes orthogonality and symmetry motifs into (a) extremal angle functionals at fixed inradius r and circumradius R ; (b) locus-based characterizations in circles; (c) orthogonal decompositions of activation/loads in applied domains; (d) geometric energy spaces where Pythagorean theorems reappear as identities under new inner products; and (e) information-geometric identities linking kinematics to statistical overlap.

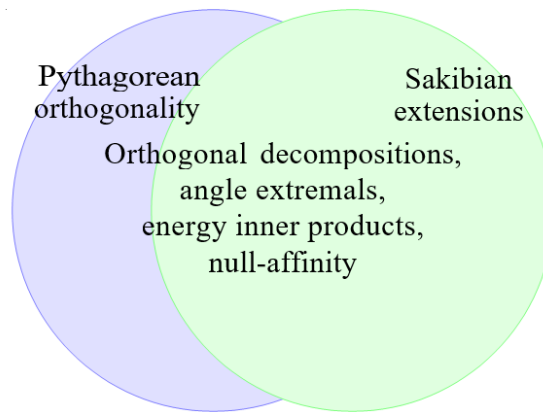


Figure 1.1: Comparative landscape: classical vs. Sakibian orthogonality.

CHAPTER 2

PYTHAGOREAN GEOMETRY REVISITED

Pythagoras' theorem $a^2 + b^2 = c^2$ encapsulates right-angle structure and inspires orthogonal decompositions in linear spaces. Two motifs recur:

1. Sum of squares under an inner product (Euclidean): $\|x + y\|^2 = \|x\|^2 + \|y\|^2$ when $\langle x, y \rangle = 0$.
2. Characterizations of rightness (e.g., via medians and altitudes).

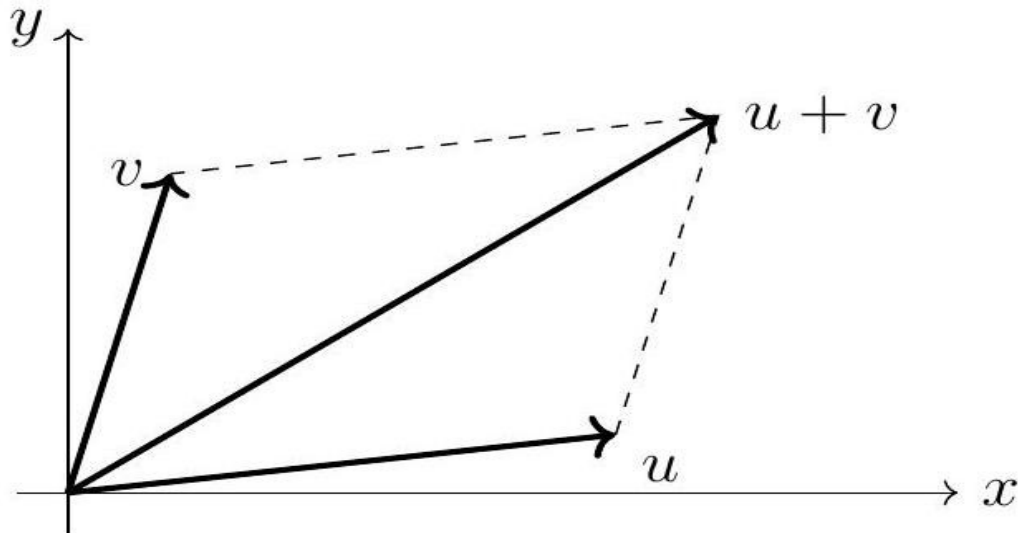


Figure 2.1: Euclidean Pythagoras: $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ when $\langle u, v \rangle = 0$.

CHAPTER 3

SAKIBIAN GEOMETRY I: NONLINEAR EXTREMALS IN TRIANGLE ANGLE SPACE

Consider triangles with fixed r and R . The symmetric functional

$$P(A, B, C) = \sum_{\text{cyc}} \left(\log \tan \frac{A}{2} \right)^2$$

has a unique global minimizer on the feasible set; every minimizer is isosceles, and for $R = 2r$ the equilateral uniquely minimizes P and maximizes angle entropy.

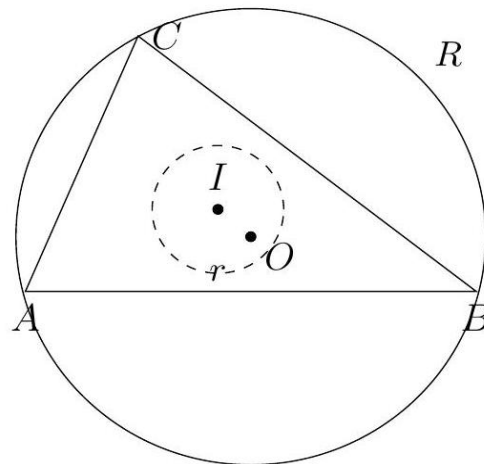


Figure 3.1: Triangle with incenter I and circumcenter O ; extremals at fixed r/R yield isosceles minimizers.

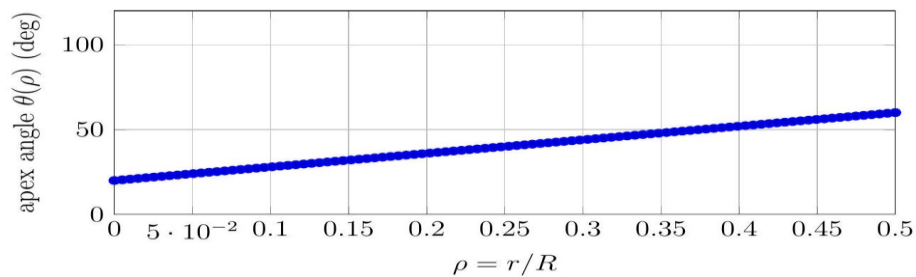


Figure 3.2: Stylized isosceles apex angle $\theta(\rho)$ rising to 60° as $\rho \rightarrow 1/2$ (equilateral).

CHAPTER 4

SAKIBIAN GEOMETRY II: THE CHORD-MIDPOINT CIRCLE (CMC)

Fix a circle ω with center O and a point X . As a line through X varies, the midpoints of the intercepted chords lie on the circle with diameter OX : the Chord-Midpoint Circle (CMC).

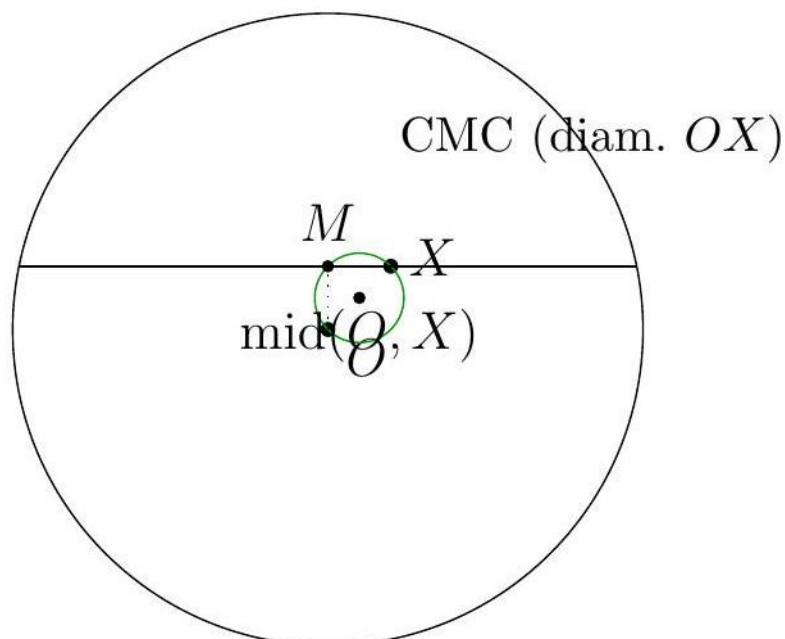


Figure 4.1: Chord-Midpoint Circle (CMC): midpoints of chords through X lie on the circle with diameter OX .

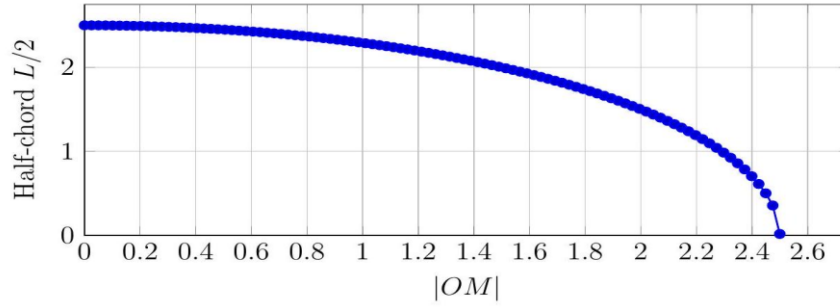


Figure 4.2: Transfer law: $(L/2)^2 + |OM|^2 = R^2$.

CHAPTER 5

SAKIBIAN GEOMETRY III: ORTHOGONAL CONTROL AND ENERGY GEOMETRIES

SOCT: ORTHOGONAL DECOMPOSITIONS IN BIOMECHANICS

Let f be muscle forces and $R^T f = \tau$ joint torques. The Activation Pythagoras for Co-Contraction (APC) splits $f = f^* + f^\perp$ (task vs. co-contraction) in a weighted inner product, so $\mathcal{C}(f) = \mathcal{C}(f^*) + \mathcal{C}(f^\perp)$.

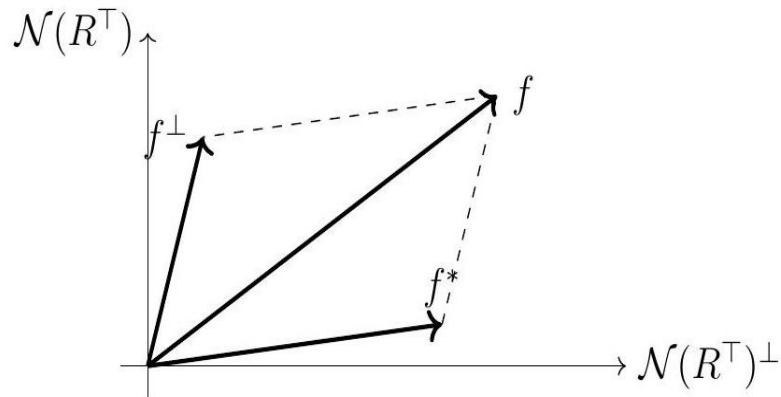


Figure 5.1: APC: $f = f^* + f^\perp$ with orthogonality in a W -inner product; cost splits Pythagorean-style.

SEG: STRUCTURAL ENERGY GEOMETRY

Given stiffness $K > 0$, define $\langle f, g \rangle_K = f^T K^{-1} g$. Then

$$\|f + g\|_K^2 = \|f\|_K^2 + \|g\|_K^2 \text{ iff } \langle f, g \rangle_K = 0$$

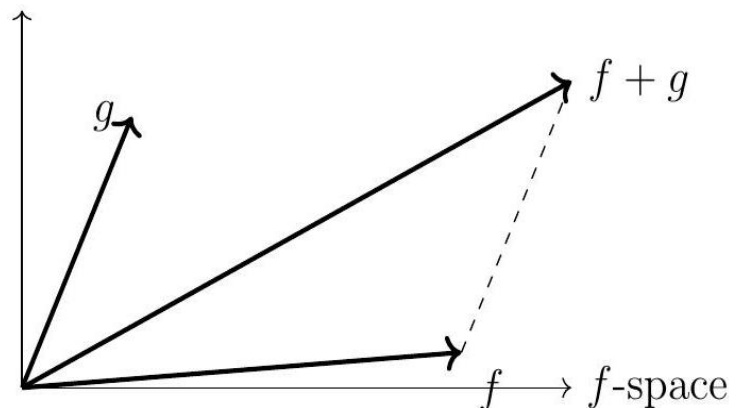


Figure 5.2: Energy inner-product geometry: $\|f + g\|_K^2 = \|f\|_K^2 + \|g\|_K^2 + 2\langle f, g \rangle_K$.

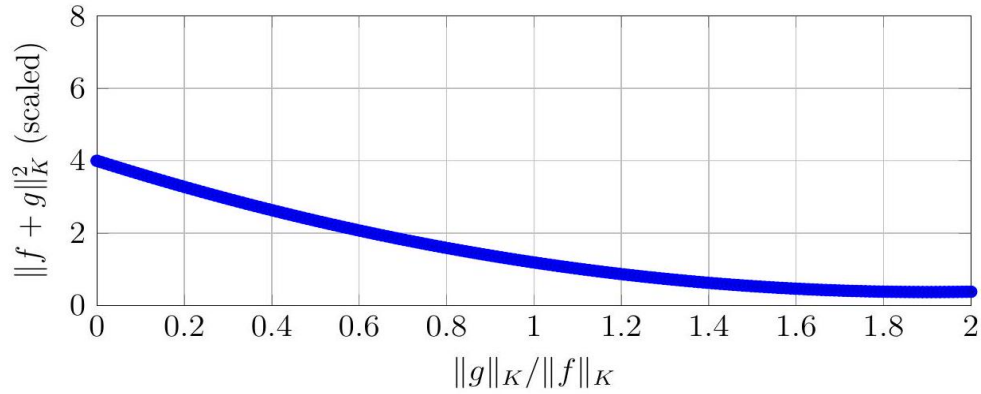


Figure 5.3: Energy cosine law illustration (fixed $\|f\|_K = 2$, angle 60°).

CHAPTER 6

SAKIBIAN GEOMETRY IV: INFORMATION-GEOMETRIC TIME DILATION

For a timelike worldline with rapidity $\theta(\tau)$, define null integrals $u = \int e^{-\theta} d\tau$, $v = \int e^{\theta} d\tau$. With measures $d\mu_+ = e^{\theta} d\tau/v$ and $d\mu_- = e^{-\theta} d\tau/u$, the Hellinger affinity equals the aging ratio

$$A(\mu_+, \mu_-) = \int \sqrt{d\mu_+ d\mu_-} = \frac{\tau}{\sqrt{uv}} = \frac{\tau}{\tau_{\text{geo}}},$$

and $B = -\log A = \log(\tau_{\text{geo}}/\tau)$ quantifies the deficit.

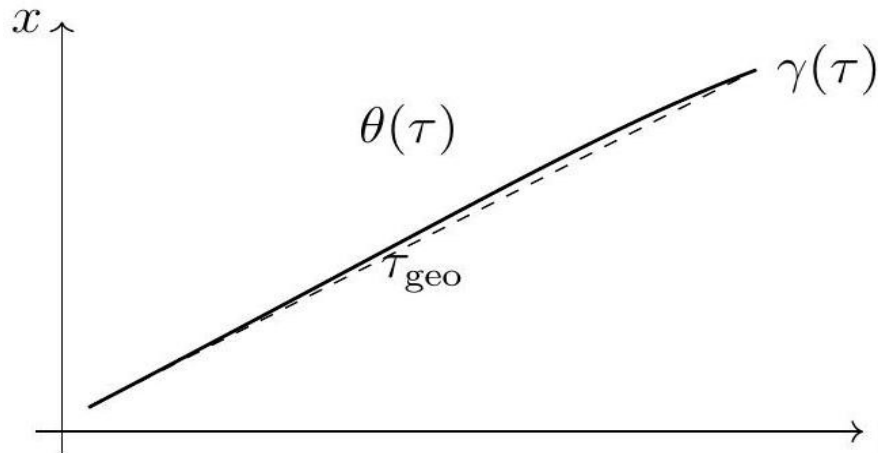


Figure 6.1: Worldline vs. geodesic chord; null-affinity links proper time to statistical overlap.

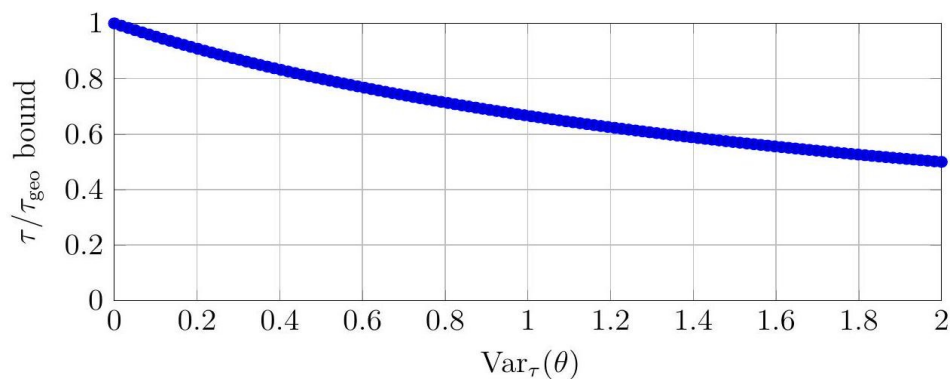


Figure 6.2: Variance bound: increasing rapidity variance tightens the upper bound on τ/τ_{geo} .

CHAPTER 7

SAKIBIAN GEOMETRY V: MEDIAN-HYPOTENUSE AND ALTITUDE RECIPROCITY

For $\triangle ABC$ with medians m_a, m_b, m_c and side c opposite C ,

$$\triangle ABC \text{ right at } C \Leftrightarrow m_a^2 + m_b^2 = m_c^2 + c^2.$$

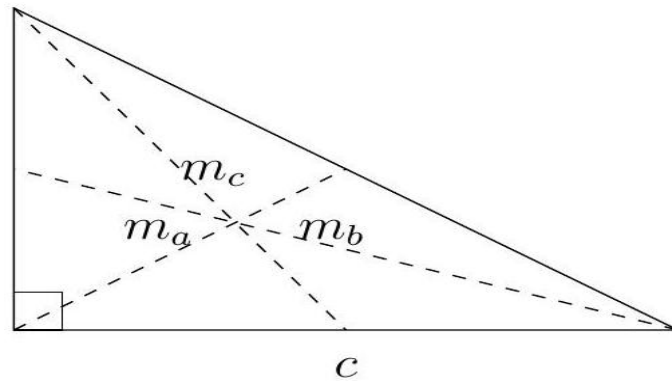


Figure 7.1: Median-Hypotenuse Pythagoras: rightness via medians and hypotenuse.

CHAPTER 8

SAKIBIAN GEOMETRY VI: ALTITUDE RECIPROCITY TRIPTYCH

With altitudes h_a, h_b, h_c , inradius r , circumradius R and sides a, b, c ,

$$\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r}, \quad \frac{\cos A}{h_a} + \frac{\cos B}{h_b} + \frac{\cos C}{h_c} = \frac{1}{R}, \quad \frac{\sin A}{h_a} + \frac{\sin B}{h_b} + \frac{\sin C}{h_c} = \frac{a^2 + b^2 + c^2}{abc}.$$

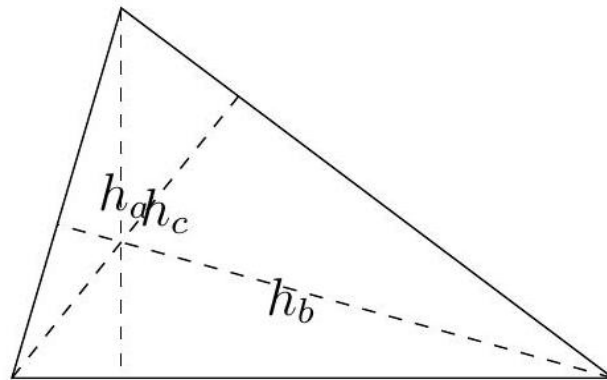


Figure 8.1: Altitudes and reciprocity with r and R .

CHAPTER 9

SAKIBIAN GEOMETRY VII: A UNIVERSAL QUADRATIC ENERGY ON RANDOM TRIANGLE POINTS

For a uniformly random interior point P of $\triangle ABC$ with perpendicular distances (d_a, d_b, d_c) to the sides and opposite side lengths (a, b, c) , define

$$F(P) = (ad_a)^2 + (bd_b)^2 + (cd_c)^2$$

Then F/Δ^2 has triangle-independent distribution with support $[4/3, 4)$, mean 2, and variance $4/15$; it is minimized at the centroid.

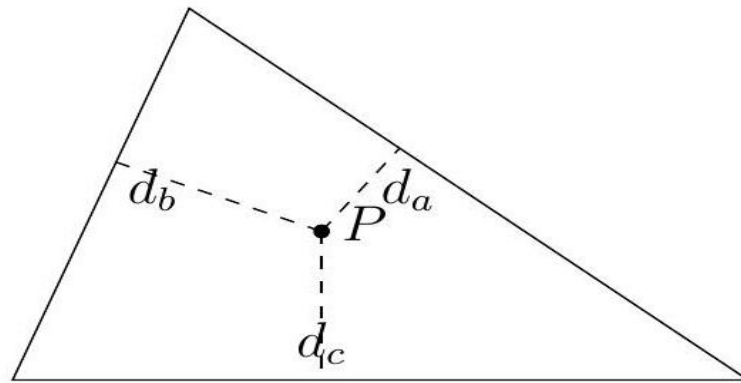


Figure 9.1: Random-point quadratic energy: $F = (ad_a)^2 + (bd_b)^2 + (cd_c)^2$.

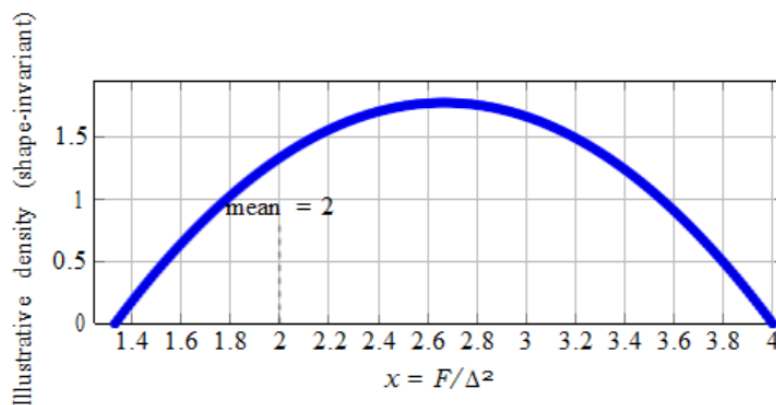


Figure 9.2: Schematic, triangle-independent law on $[4/3, 4)$ with mean 2 and variance $4/15$.

CHAPTER 10

UNIFICATION AND COMPARATIVE THEOREMS

We collect the motifs:

- Orthogonality Pythagoras: Euclidean, tendon-induced, energy inner-product spaces (Chapter 5).
- Entropy/Schur-concavity: \rightarrow extremals in angle space at fixed r/R (Chapter 3).
- Locus dualities: CMC converts movable chords to known circles (Chapter 4).
- Information overlap: null-affinity encodes proper time (Chapter 6).
- Rightness classifiers: medians and altitudes (Chapters 7 and 8).
- Affine invariants: random-point quadratic energy (Chapter 9).

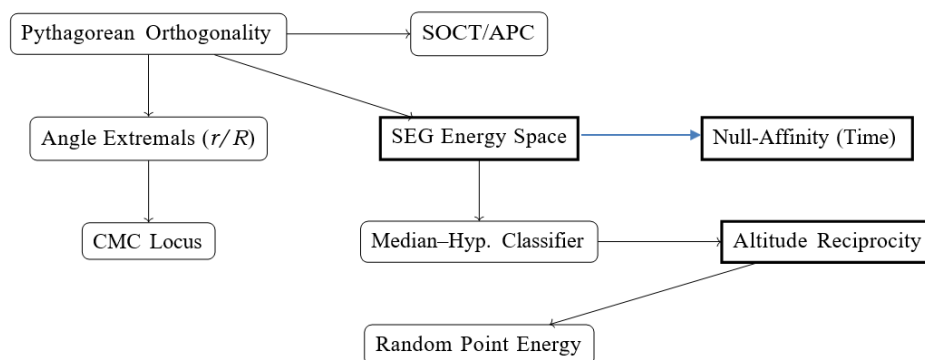


Figure 10.1: A unified map of Pythagorean \rightarrow Sakibian motifs across spaces.

CHAPTER 11

APPLICATIONS AND CASE STUDIES STRUCTURAL/CLINICAL ANALYTICS

- Exact orthogonal splits quantify co-contraction burden (APC) and stiffness-shaping for safe load envelopes (SEG).
- Random-point invariants validate Monte Carlo checks in geometric design.

GEOMETRIC PROBLEM SOLVING

- CMC simplifies center-recovery and chord families.
- Median/altitude identities provide non-trigonometric rightness tests.

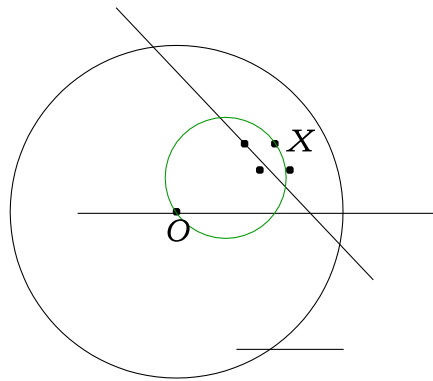


Figure 11.1: Regular fan: equally spaced chords through X give midpoints forming a regular polygon on the CMC.

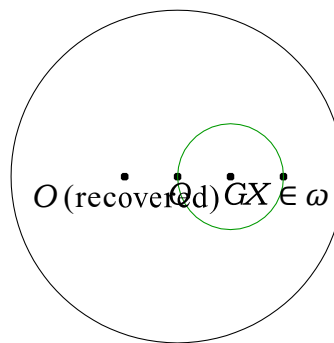


Figure 11.2: Center recovery: midpoints on the CMC give its center $G = \text{mid}(O, X)$; reflect X across G to recover O .

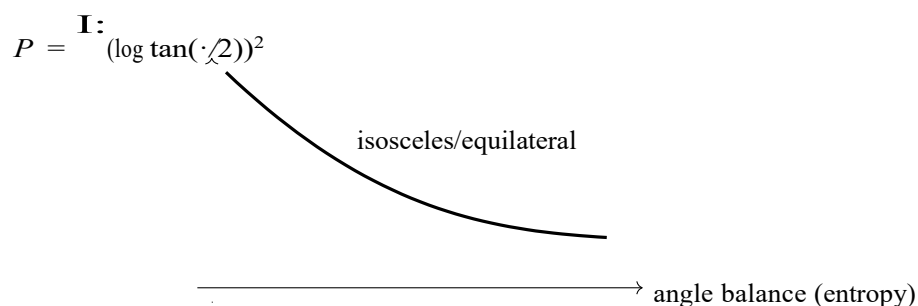


Figure 11.3: Schematic: nonlinear functional P decreases as the angle-triple balances.

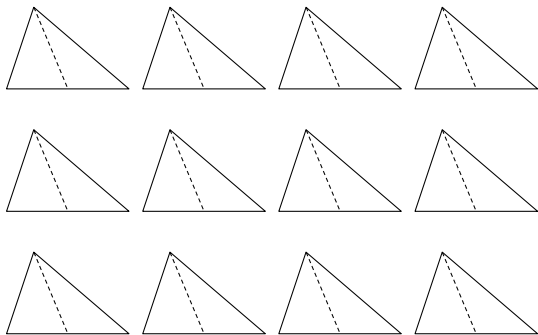


Figure 11.4: Mini-gallery of triangle schematics (medians, altitudes, chords).

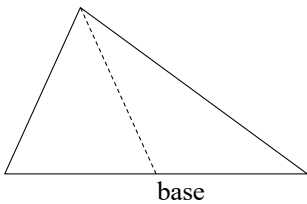


Figure 11.5: Median from apex to base midpoint.

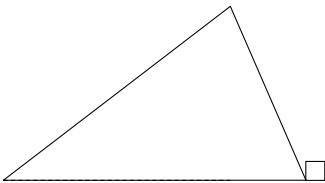


Figure 11.6: Right triangle with altitude to hypotenuse.

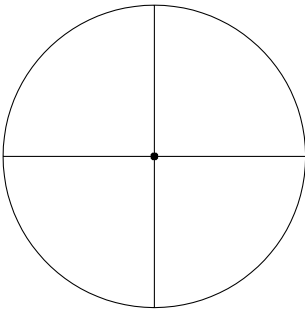


Figure 11.7: Orthogonal diameters partitioning the circle.

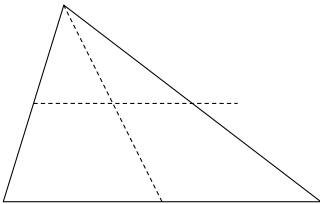


Figure 11.8: A cevian and a parallel through its midpoint.

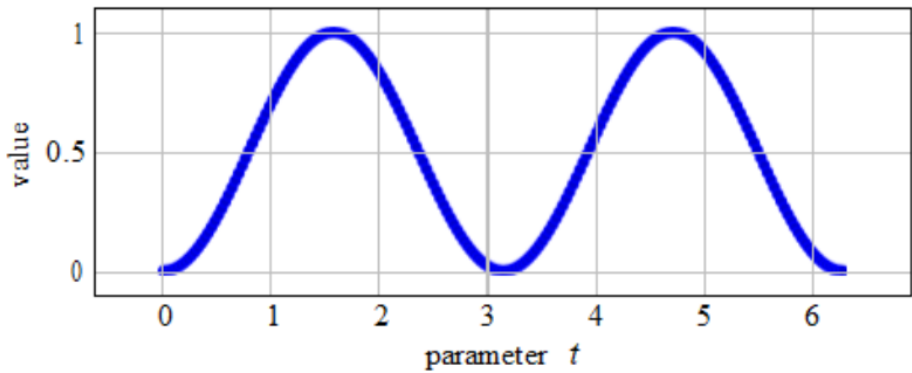


Figure 11.9: Illustrative squared-sine shape used as a generic energy profile.

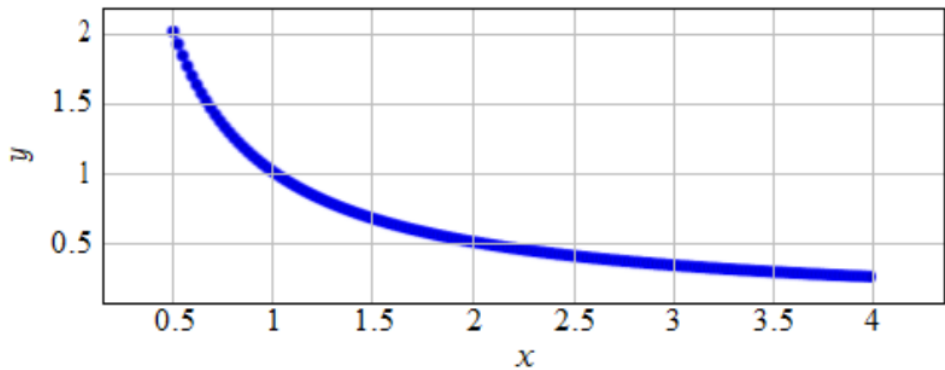


Figure 11.10: Reciprocity schematic ($y = 1/x$).

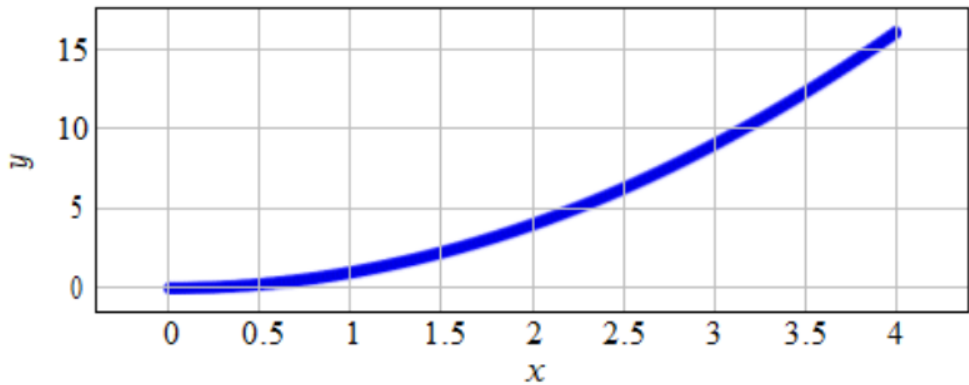


Figure 11.11: Quadratic growth motif.

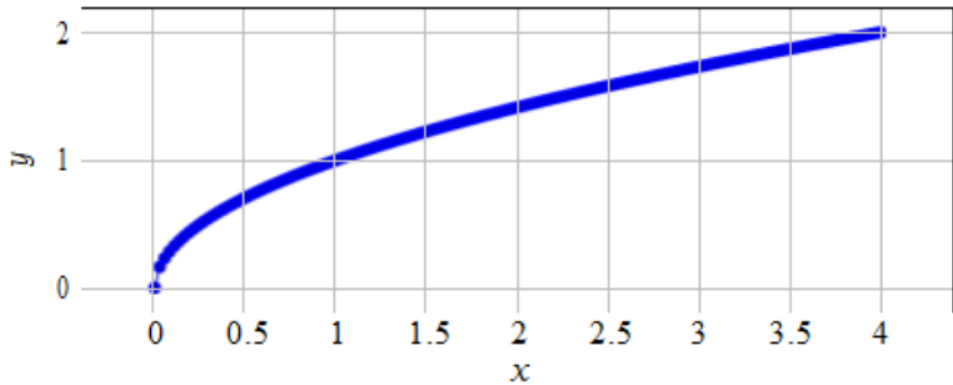


Figure 11.12: Square-root response motif.

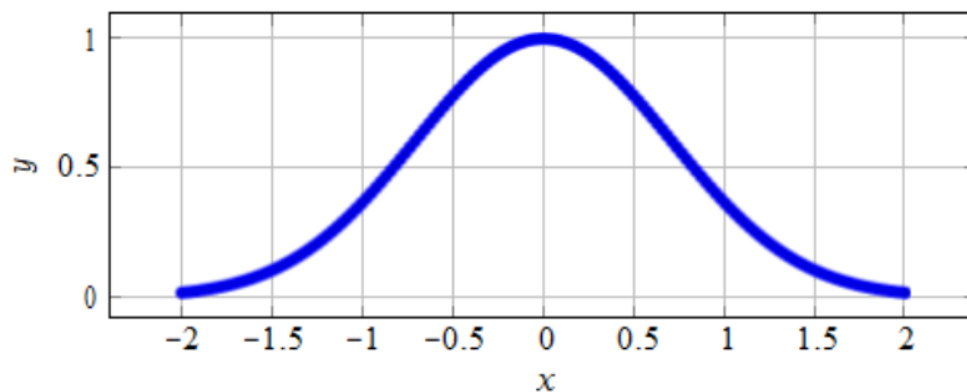


Figure 11.13: Gaussian-like schematic (for statistical overlap intuition).

CHAPTER 12

CONCLUSION

Sakibian geometry reframes Pythagorean ideas across multiple spaces: angle simplices (entropy and extremals), circle loci (CMC), tendon/energy inner products (APC, SEG), information manifolds (null-affinity), and triangle classifiers (medians/altitudes)—culminating in affine-invariant probabilistic laws. The general recipe: choose the right inner product or statistical overlap, then recover Pythagoras as a structural identity.

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