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GRAVITATIONAL LENSING IN GENERAL RELATIVITY

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Abstract

Einstein's groundbreaking formulation of General Relativity fundamentally predicted the phenomenon of gravitational lensing, which was empirically confirmed in 1919 during the solar eclipse expedition. Over subsequent decades, some of the brightest minds in astrophysics and theoretical physics advanced the understanding of this effect, exploring its various facets and potential applications. Early theoretical investigations considered gravitational lensing as a natural cosmic telescope capable of magnifying and resolving extremely faint and distant objects, thereby opening new windows into the early universe. Researchers also examined the probability of multiple or Einstein ring-like images forming when light from background sources passes near massive foreground objects, providing unique insights into mass distributions and spacetime geometry. Additionally, gravitational lensing emerged as a powerful method for determining cosmological parameters, notably offering a means to measure the Hubble constant with unprecedented precision. The field transitioned from purely theoretical pursuits to observational science following the discovery of the first doubly imaged quasar in 1979, which provided concrete evidence of gravitational lensing at extragalactic scales. Since then, numerous phenomena such as Einstein rings, luminous arcs, galactic microlensing events, and weak gravitational lensing have been observed and studied extensively. These phenomena have proven to be invaluable tools in astrophysics, yielding a wide array of scientifically significant results. For instance, gravitational lensing has been instrumental in mapping the large-scale distribution of matter in galaxies and clusters, refining the cosmic distance scale, and probing the structure and evolution of quasars. Most notably, it has played a crucial role in unveiling the presence and distribution of dark matter within galactic halos and has contributed to our understanding of the nature of dark energy. The remarkable successes achieved through gravitational lensing over recent years underscore its status as one of the most versatile and profound astrophysical tools. Looking ahead, ongoing advancements in observational technology and theoretical modeling promise to unlock even more astonishing discoveries, solidifying gravitational lensing's role as a cornerstone of modern cosmology and astrophysics. As our understanding deepens, gravitational lensing is poised to continue revealing the universe's most fundamental mysteries with unprecedented clarity.

Key words : Gravitational Lensing, Dark Matter, Cosmological Parameters, Einstein Rings, Extragalactic Astronomy, Cosmic Microwave Background

Introduction

According to Einstein's General Theory of Relativity, the professed force of gravitation is only a superficial force, proposed by our method of measuring space and time using properties of Euclidean geometry, while in reality, the space-time continuum in the astronomical body's gravitational field follows Non-Euclidian laws of geometry or curved geometry.

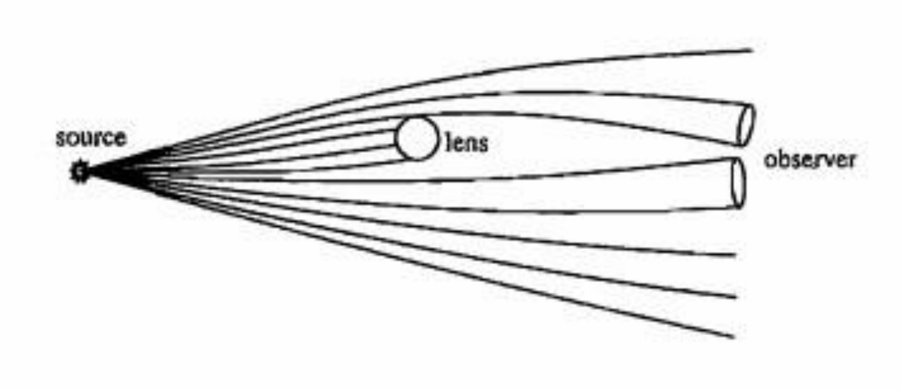

Figure 1: Gravitational Lensing

Gravitational lensing is the astrophysical phenomenon whereby the propagation of light is affected by the distribution of mass in the universe. As photons travel across the universe, their trajectories are perturbed by the gravitational effects of mass concentrations with respect to those they would have followed in a perfectly homogeneous universe. The photon seems to follow its geodesic (normal path). A Geodesic is the shortest path between two points in differential geometry (here, in curved spacetime). Gravitational lensing works in an similar way to the description of the propagation of light through normal lenses such as a pair of spectacles in which light bends due to refraction and focuses into the eye, whereas in gravitational lenses light bends around a concentration of mass (usually galaxies, stars, black holes) and refocuses somewhere else. The effects can be very strange and very strong; considering near highly dense masses. The light coming from lensed galaxies and clusters are stretched into arcs as the light passes close to the foreground cluster. Gravitational lensing is a "relatively" new field which has been gained astronomical society's curiosity recently. It is most appreciated for its beautiful and robust appeal, to study an impressive range of astrophysical phenomena, from galaxies to clusters, to black holes, dark energy and dark matter.

Newtonian and GR predictions

The current idea of the deflection of light comes from Einstein's General Theory of Relativity, though hints of the idea can be seen in Newton’s book Opticks[1]. One of the queries stated “Do not Bodies act upon Light at a distance, and by their action bend its rays, and is not this action (cæteris paribus) strongest at the least distance?”, but since he thought of light as only a wave phenomenon, therefore he did not know how to explain the deflection accurately. In fact, this idea is not so much revolutionary, because on the basis of the corpuscular theory of light, and Newton’s laws of mechanics and gravitation, it is easy to conjecture that corpuscles can deviate slightly when it passes near a highly dense body, assuming that gravitational acceleration affect corpuscles similarly to particle matter.

Newtonian gravitational deflection

In 1783, hypothesizing that light consists of corpuscles, John Mitchell (1724-1793) wrote a paper on a technique to measure the mass of stars by detecting the modification in the light speed by the gravitational pull as the light corpuscles perturbed in star's gravitational field.[2]

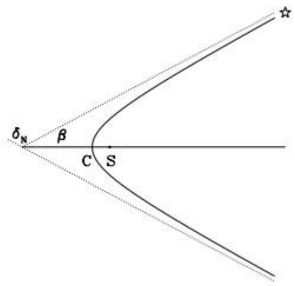

Figure 2: The light ray follows a hyperbolic orbit about the source andthe distance of the closest approach is r_s

From Fig.2, The speed of light is so large that it beats the escape velocity. Thus, the resulting orbit will be a hyperbolic orbit. For a hyperbolic orbit of a light particle of mass '*m*' in a Newtonian gravitational field around a central mass *M*, the eccentricity of the boundless hyperbolic orbit is given by,

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{G^2 m^3 M^2}} \ldots\ldots 1$$

The constants of motion E and L are easily assessed at point *C*. Let *v* is the total velocity of the particle, v_t is the tangential component of velocity and *r* is the radial distance from the center of the mass.

Then,
Total Energy, *E*

$$E = \frac{mv^2}{2} - \frac{GMm}{r} \ldots\ldots 2$$

Angular momentum, *L*

$$L = mrv_t$$

Let r_p be the impact perimeter or the closest approach *C* (the perihelion) of the particle to the dense body, at which $v = v_t$
Then,

$$\varepsilon = \sqrt{1 + \left(\frac{r_p v_t^2}{GM}\right)} - \frac{2r_p v_t^2}{GM} = 1 - \frac{r_p v_t^2}{GM} \ldots\ldots 3$$

Now, we consider v_t=*c* at the closest approach *C* The angle *β* of the asymptote to the hyperbole of eccentricity *ε* is given by

$$\cos\beta = \frac{1}{e}$$

The total angular deflection of the beam of light is which for small angles *β* and for *M* much less than r_p is given in Newtonian mechanics by

$$\delta_{NG} = \pi - 2\beta$$

$$\delta_{NG} = \pi - 2cos^{-1}\left(\frac{1}{\varepsilon}\right) = \pi 2cos^{-1}\left(\frac{GM}{GM - c^2 r_p}\right) 2\frac{GM}{r_s c^2} \ldots\ldots\ldots\ldots\ldots\ldots.4$$

The result is exactly the same as achieved with modern space-time curvature calculations except for a factor of two. When in 1911, Einstein first calculated the gravitational bending of light, using the Equivalence Principle and the equivalent mass-energy of a photon[3]. The calculation yielded $\boldsymbol{\delta_{NG}}$.

Only in his second calculation, when he completed his General Theory of Relativity, which considered the effects of spacetime curvature, he obtained a value twice as large as his first calculation, i.e., $\boldsymbol{\delta_{GR}}$**.**

$$\delta_{GR} = 4\frac{GM}{r_s c^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..5$$

Characteristics of Gravitational Lensing

In order to get the correct value of the deflection of light by a mass M, we used the Theory of General Relativity. According to this theory, the deflection is described by geodesic lines following the curvature of the space-time. In curved spacetime, geodesic lines are lines which are as "straight as possible", resembling straight lines in flat space-time. As a light ray follows the curvature, it is bent towards the mass which causes the space-time to be curved.

$$\delta_{GR} = 4\frac{GM}{r_s c^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.6$$

Where, *G, M, b, c* are the gravitational constant, mass of the object, speed of light and the impact parameter or distance of the closest approach respectively.

This bending gives rise to several important phenomena:

1. The light ray can arrive at the eyes of the observer traveling from multiple paths around the mass M. The observer, will see an image of the source along the tangent of the undeflected ray at his position and will see multiple images of a single source.

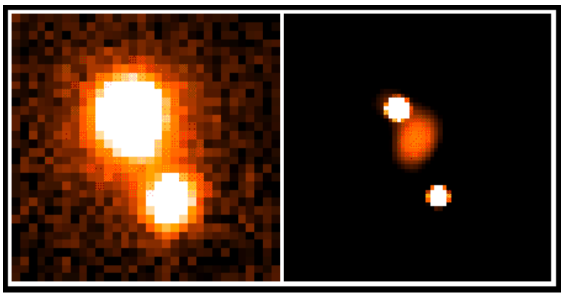

Fig 3. Left panel shows two images of double quasar HE 1104-1825. Right Panel shows the galaxy lensing the quasar.

2. Moreover, light deflection of two nearby rays may be different. Suppose two rays adjacent to each pass near a dense mass distribution. The deflection of the rays depends on the radial distance between the deflector and light ray, therefore the ray near the deflector follows a geodesic different to another ray and gets bent greater, thus the source will appear stretched. It is thus expected that gravitational lensing will typically distort the sources.

3. The lensed source can be magnified or shrinked since the size of the source is not conserved in gravitational lensing. But since photons can neither be created nor destroyed, the surface brightness of the source is preserved. But the distortion results in an image which covers more surface area than the original source did, so this results in an overall brightening.

Fig 4. Abell 1689 a "2 million year wide" lens distorting various galaxies behind it.

4. Light rays arriving in the collimated beam and passing at a given distance either side of the star are brought together or 'focused'. The focal length of an ordinary lens is defined as the distance from the lens at which a collimated incident beam is brought to a focus. Using this method we found out the focal length of a gravitational lens to be: The focal length is a function of the radial distance b from the center of mass. The ray incident at a small radius is brought to focus closer to the star than the rays incident at a large radius.
5. Using the 'thin lens formula' $1/d_1+1/d_2=1/f$, where d_1, d_2 are the source lens and lens image distances respectively, when we solve for b we get, Einstein Radius,

$$b_E = \sqrt{4\frac{GM}{c^2}\frac{(d_1+d_2)}{d_1 d_2}} \quad \ldots\ldots\ldots\ldots 7$$

We get a ring of the light called *'Einstein ring'* of the source which is observed by the receiver, when the source, lens, and receiver are aligned. Generally, this alignment is imperfect and we get a pair of images smeared into arcs.

Types of Lensing

Strong Lensing

Strong lensing happens when the source, the lens, and the observer are arranged sufficiently well and close, and the bending angle is large enough to allow the different images to be resolved. This results in huge distortions in the images producing arcs, Einstein rings and multiple images of the source. It requires a lens of mass density greater than some critical density. This type of lensing usually occurs in the central regions of galaxies and clusters.

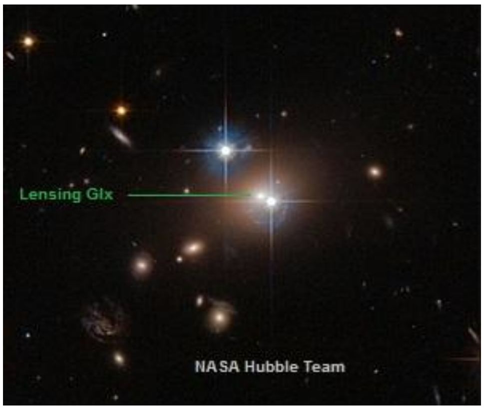

Fig 5. The Twin Quasar Q0957

The first strong lensing observation was seen in 1979 by Walsch, Carswell, and Weymann of the doubly imaged Q0957+561 Quasar. A pair of two blue dots were found within the parameter of the quasar separated by ~6 arcseconds. This observation was a rather accident as told by Walsch. In the beginning, it was uncertain whether this image was the consequence of space-time curvature and was an illusion to the observer or these quasars were twins. But soon enough, the two blue dots were observed to have the same spectra from the telescope. Moreover, later similar light curves of the image proved the system was the first visible example of Gravitational lensing.

In 1986, Lynds and Petrosian found huge gravitationally lensed luminous arcs in the massive Abell 370 clusters at KPNO(Kitt Peak National Observatory). These huge arcs are the result of the huge distortion due to the lensing effect of the foreground massive clusters on background galaxies, with huge magnifications that can distort the galaxy shapes into long arcs around the clusters' cores.

Strong gravitational lensing also may provide information for cosmology. For example, measurement of Hubble constant can be obtained using the time-delay among the multiple images of a lensed quasar.

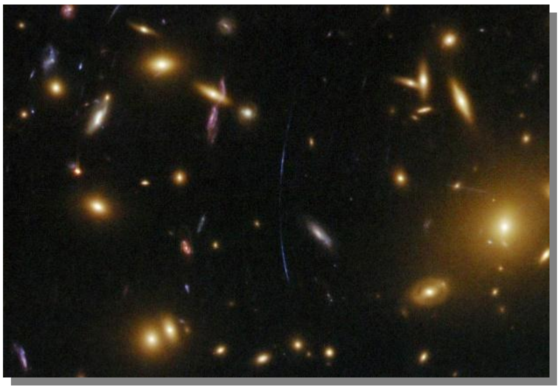

Fig 6. Giant Luminous Arc in Abell 370

Until recently, the general objects to study gravitational lensing were massive galaxy clusters and galaxies. But, Galaxy groups are comprised of galaxies and have often low density than compared to clusters, making them more challenging to detect. The study of lensing of these intermediate-scale structures could lead to a better understanding of the evolution of structures in the universe.

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Weak Lensing

In distinction to the phenomenon of strong lensing, weak lensing deals with the distortion of light that is measured statistically and not separately. As we have discussed earlier, "Strong lensing" phenomenon involves the formation of multiple images and high magnification when the observer and the source are aligned properly -which is a rare phenomenon[16]. But, weak lensing is rather much more common. Since each light ray path is affected by matter inhomogeneities along its path, it is just a matter of how accurate we can measure the lensing.

Weak lensing by foreground galaxy clusters creates small distortions in the shapes of background galaxies. By statistically averaging these small distortions, mass estimates of the clusters can be obtained. Most of what is understood about the distribution of mass in the universe come from observing the distribution of galaxies. We can determine how well the light from galaxies traces the mass by observing the mass of its constituents. Cosmologists are interested in weak lensing by the generic large-scale structure of the universe. In this case, suggesting the goal is not to measure the distribution of dark matter. The main goal is to compare the observations of distortions of galaxy images to the underlying spectrum of mass power.

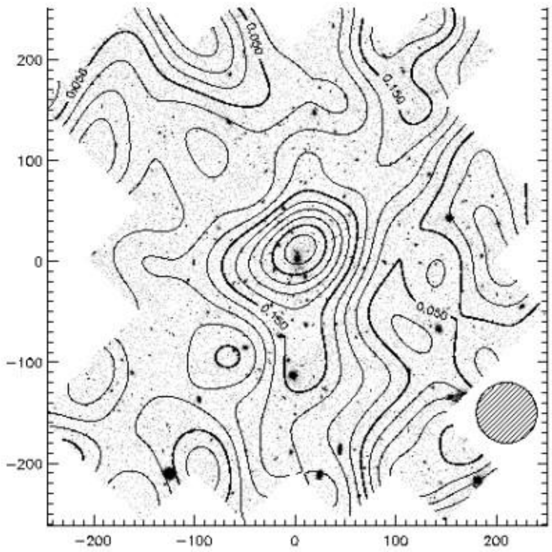

Fig 7. The reconstructed mass distribution of cluster CL1358+62

The first observation of coherent weak lensing of distorted background galaxies was measured in 1990 around the galaxy clusters Abell 1689 and CL1409+52.

Microlensing

Microlensing is the type of gravitational lensing in which the gravitational lensing effect is small-scale. The gravitational field of the lensing object is not strong enough to form distinct images of the background source. In some observations, the lensing is of an image that is so small or faint that instead of observing multiple images, the additional light bent towards the observer making the source appears brighter. The surface brightness remains constant but as numerous images of the object appear, the object appears brighter and bigger. The deflection of the source is typical of the order of μ arcsecs[14,15].

Due to the small size of the source which would have been too dim to be visible, this lensing can have effects in many measurements.

It has been known for a very long time that halos of galaxies must carry some kind of dark matter. Many different particles/objects had been suggested as constituents of this halo dark matter. One of the most proposed candidates are the brown dwarfs (failed stars), which due to lack of the process fusion appears dark. These objects are certain to exist, we just do not know how many there are. Subsequently, this type of dark matter candidate was labeled as 'MACHOS (MAssive Compact Halo Object)'. As MACHOS due to their size and visibility cannot be seen directly, their gravitational lensing effects can be observed when they pass in front of a source (e.g. a star, planet nearby), they can cause the star to become brighter for a while.

Conclusion

Gravitational lensing stands as one of the most extraordinary and insightful phenomena in modern astronomy. Its theoretical foundations, rooted in Einstein's General Theory of Relativity, have not only been confirmed through numerous observations but have also transformed into a vital tool for exploring the universe's unseen components. The phenomena of multiple images, Einstein rings, microlensing of quasars and stars, as well as weak lensing distortions, were predicted long before their empirical detection. These phenomena are fundamentally rooted in the simple yet powerful geometrical principle that mass curves spacetime, bending the path of light passing nearby.

The simplicity of the core concept light following curved geodesics in a warped space time belies the profound impact that gravitational lensing has on our understanding of cosmic structures. It enables astronomers to perform qualitative estimates and precise quantitative calculations of mass distributions in various astrophysical objects. Since the deflection of light depends on the total gravitational influence along the line of sight, gravitational lensing provides a direct measure of the total mass both luminous and dark that resides in galaxies, galaxy clusters, and large-scale structures.

Probing Dark Matter and Cosmology

One of the most significant contributions of gravitational lensing is its unique ability to probe dark matter. Unlike luminous matter, dark matter does not emit, absorb, or reflect electromagnetic radiation, making it invisible through traditional telescopic observations. However, because gravitational lensing responds solely to mass, it offers an indispensable method to map the distribution of dark matter in galactic halos, galaxy clusters, and the cosmic web. This ability has led to substantial breakthroughs in understanding the large-scale structure of the universe and testing cosmological models.

Furthermore, lensing phenomena serve as critical tools for studying galaxy evolution. Strong lensing systems magnify distant galaxies, allowing detailed observations of their morphology, star formation rates, and other properties that would otherwise be inaccessible due to their faintness and small angular size. Such studies shed light on how galaxies form and evolve over cosmic time, helping to refine models of galaxy assembly and the influence of environment.

Current Status and Technological Advancements

The past few decades have witnessed remarkable progress in the observation and analysis of gravitational lensing phenomena. Large-scale surveys, such as the Sloan Digital Sky Survey (SDSS), the Dark Energy Survey (DES), and upcoming projects like the Vera C. Rubin Observatory's Legacy Survey of Space and Time (LSST), Euclid, and the Nancy Grace Roman Space Telescope, are expected to discover thousands of new lensing systems. These surveys will provide high-quality data across vast regions of the sky, enabling statistical studies of lensing effects and their cosmological implications.

Advances in high-resolution imaging, spectroscopy, and computational modeling have significantly improved our ability to interpret lensing signals. Sophisticated software tools now allow detailed reconstruction of mass profiles and three-dimensional mass distributions. The integration of machine learning techniques further accelerates the detection and classification of lensing events, especially in the era of big data.

Challenges and Limitations

Despite these advances, several challenges hinder the full exploitation of gravitational lensing as a cosmological probe. One major issue is the degeneracy in mass modeling different mass distributions can produce similar lensing signatures, complicating the inference of precise mass profiles. The mass-sheet degeneracy, for example, can lead to uncertainties in the determination of the true mass distribution.

Systematic uncertainties in measurements, such as shape measurement errors in weak lensing, contamination from line-of-sight structures, and assumptions about the mass profiles, can also limit accuracy. Additionally, the complexity of strong lensing systems often involving multiple lens components and complex geometries demands computationally intensive modeling efforts.

Future Prospects and Opportunities

Looking ahead, the future of gravitational lensing research is promising. The next generation of observational facilities will dramatically increase the quantity and quality of lensing data. Precise measurements of the Hubble constant (H_0) through time-delay cosmography in strongly lensed quasars are expected to resolve current tensions in its value, which has significant implications for our understanding of the universe's expansion rate.

Mapping dark matter halos across cosmic time will become more detailed, revealing the growth and evolution of structure. The synergy between lensing and other cosmological probes, such as galaxy clustering, cosmic microwave background (CMB) measurements, and supernova observations, will facilitate a comprehensive understanding of fundamental physics, including the nature of dark energy and potential modifications to General Relativity.

Moreover, future surveys will enable the detection of rare and exotic lensing phenomena, such as Einstein crosses, lensing of gravitational waves, and microlensing events caused by primordial black holes or other compact objects. These phenomena could offer new insights into the early universe, the nature of dark matter, and potential physics beyond the Standard Model.

Implications for Cosmology and Fundamental Physics

Gravitational lensing remains an indispensable tool in constraining key cosmological parameters. Its capacity to measure the distribution of matter, both luminous and dark, across cosmic scales makes it central to understanding the history and fate of the universe. The precise determination of the Hubble constant through lensing time delays can help resolve current discrepancies between local and early-universe measurements. Mapping dark matter halos will test predictions of structure formation models and the nature of dark matter particles.

Furthermore, gravitational lensing provides an experimental arena for testing the foundations of gravity itself. Deviations from the predictions of General Relativity, if observed, could point toward new physics. As such, lensing studies are at the forefront of efforts to explore fundamental questions about the universe's fabric.

In conclusion, gravitational lensing exemplifies the profound interplay between theoretical physics, observational astronomy, and computational analysis. Its discovery and subsequent development have revolutionized our comprehension of the cosmos, enabling direct visualization and mapping of the universe's unseen matter and structures. The phenomena of multiple images, Einstein rings, microlensing, and weak lensing are not merely astrophysical curiosities but are powerful probes of the universe's composition, evolution, and fundamental laws.

As technological advancements continue and new observational facilities come online, the potential of gravitational lensing to answer pressing cosmological questions will only grow. It offers the promise of more precise measurements of the universe's expansion rate, detailed maps of dark matter, insights into galaxy formation, and tests of physics under extreme conditions. Harnessing this potential will require continued collaboration across disciplines, refinement of models, and innovative analytical techniques.

With these efforts, gravitational lensing will undoubtedly remain a cornerstone of modern cosmology, guiding us toward a deeper understanding of the universe's origin, structure, and destiny. It embodies the elegance of Einstein's insight that gravity, through the curvature of spacetime, shapes the very fabric of our universe and illustrates how that understanding continues to illuminate the cosmos in unprecedented ways.

Recommendation on Gravitational Lensing in General Relativity

Gravitational lensing, a remarkable prediction of Einstein's General Theory of Relativity, provides a powerful observational tool for probing the universe's structure and content. It occurs when massive objects, such as galaxies or galaxy clusters, bend the fabric of spacetime, causing the deflection of light from background sources.

1. Enhanced Theoretical Models: Develop and refine theoretical models that accurately describe light deflection in complex gravitational fields, including strong and weak lensing regimes. Incorporate higher-order corrections and account for potential contributions from dark matter and dark energy.

2. Advanced Observation Techniques: Leverage high-resolution telescopes and surveys (e.g., LSST, Euclid, JWST) to detect and analyze gravitational lensing events with greater precision. Emphasize multi-wavelength observations to gain comprehensive insights.

3. Data Analysis and Simulation: Implement sophisticated data analysis algorithms and numerical simulations to interpret lensing phenomena. Use these to map mass distributions, including dark matter halos, and to test predictions of General Relativity at various scales.

4. Interdisciplinary Collaboration: Foster collaboration between theorists, observational astronomers, and computational scientists to enhance understanding and application of gravitational lensing phenomena.

5. Educational and Outreach Programs: Promote educational initiatives to increase awareness of gravitational lensing as a tool for cosmology, emphasizing its role in uncovering the universe's mysteries.

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